MAC 2311
Chapter 2 Review Materials (Part III)
Topics Include Related Rates, Differentials, and Linear Approximations

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

1) Suppose that the radius r and the circumference $C = 2\pi r$ of a circle are differentiable functions of t. Write an equation that relates dC/dt to dr/dt. 1) _____

A) $\frac{dC}{dt} = 2\pi r\frac{dr}{dt}$ B) $\frac{dC}{dt} = \frac{dr}{dt}$ C) $\frac{dC}{dt} = 2\pi\frac{dr}{dt}$ D) $\frac{dr}{dt} = 2\pi\frac{dC}{dt}$

2) The area of the base B and the height h of a pyramid are related to the pyramid's volume V by the formula $V = \frac{1}{3}Bh$. How is dV/dt related to dh/dt if B is constant? 2) _____

A) $\frac{dV}{dt} = B\frac{dh}{dt}$ B) $\frac{dV}{dt} = \frac{dh}{dt}$ C) $\frac{dV}{dt} = \frac{1}{3}\frac{dh}{dt}$ D) $\frac{dV}{dt} = \frac{B}{3}\frac{dh}{dt}$

3) If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, $c^2 = a^2 + b^2$. How is dc/dt related to da/dt and db/dt? 3) _____

A) $\frac{dc}{dt} = a^2\frac{da}{dt} + b^2\frac{db}{dt}$ B) $\frac{dc}{dt} = a\frac{da}{dt} + b\frac{db}{dt}$

C) $\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt}$ D) $\frac{dc}{dt} = \frac{1}{c}\left(a\frac{da}{dt} + b\frac{db}{dt}\right)$

4) The range R of a projectile is related to the initial velocity v and projection angle θ by the equation $R = \frac{v^2 \sin 2\theta}{g}$, where g is a constant. How is dR/dt related to dθ/dt if v is constant? 4) _____

A) $\frac{dR}{dt} = \frac{2v^2 \cos 2\theta}{g}\frac{d\theta}{dt}$ B) $\frac{dR}{dt} = -\frac{v^2 \cos 2\theta}{g}\frac{d\theta}{dt}$

C) $\frac{dR}{dt} = \frac{v^2 \cos 2\theta}{g}\frac{d\theta}{dt}$ D) $\frac{dR}{dt} = \frac{2v^2 \sin 2\theta}{g}\frac{d\theta}{dt}$

5) The range R of a projectile is related to the initial velocity v and projection angle θ by the equation $R = \frac{v^2 \sin 2\theta}{g}$, where g is a constant. How is dR/dt related to dv/dt and dθ/dt if neither v nor θ is constant? 5) _____

A) $\frac{dR}{dt} = \frac{1}{g}\left(4v \cos 2\theta\frac{d\theta}{dt}\frac{dv}{dt}\right)$ B) $\frac{dR}{dt} = \frac{v}{g}\left(v \cos 2\theta\frac{d\theta}{dt} + 2 \sin 2\theta\frac{dv}{dt}\right)$

C) $\frac{dR}{dt} = \frac{1}{g}\left(v \cos 2\theta\frac{dv}{dt} + \sin 2\theta\frac{d\theta}{dt}\right)$ D) $\frac{dR}{dt} = \frac{2v}{g}\left(v \cos 2\theta\frac{d\theta}{dt} + \sin 2\theta\frac{dv}{dt}\right)$

Provide an appropriate response.

6) If $xy + x = 12$ and $dx/dt = -3$, then what is dy/dt when $x = 2$ and $y = 5$? 6) _____

A) −3 B) −9 C) 9 D) 3

7) If $xy^2 = 4$ and $dx/dt = -5$, then what is dy/dt when $x = 4$ and $y = 1$? 7) _____

A) $\frac{5}{8}$ B) $\frac{8}{5}$ C) $-\frac{5}{8}$ D) $-\frac{8}{5}$

8) If $y\sqrt{x+1} = 12$ and $dx/dt = 8$, then what is dy/dt when $x = 15$ and $y = 3$? 8) _____

A) $\frac{4}{3}$ B) $-\frac{3}{4}$ C) $\frac{3}{4}$ D) $-\frac{4}{3}$

Solve the problem.

9) A company knows that the unit cost C and the unit revenue R from the production and sale of x units are related by $C = \frac{R^2}{158{,}000} + 3405$. Find the rate of change of unit revenue when the unit cost is changing by \$12/unit and the unit revenue is \$2000. 9) _____

A) \$474.00/unit B) \$120.00/unit C) \$340.50/unit D) \$407.25/unit

10) A wheel with radius 3 m rolls at 19 rad/s. How fast is a point on the rim of the wheel rising when the point is $\pi/3$ radians above the horizontal (and rising)? (Round your answer to one decimal place.) 10) _____

A) 57.0 m/s B) 28.5 m/s C) 14.3 m/s D) 114.0 m/s

11) A piece of land is shaped like a right triangle. Two people start at the right angle of the triangle at the same time, and walk at the same speed along different legs of the triangle. If the area formed by the positions of the two people and their starting point (the right angle) is changing at 3 m^2/s, then how fast are the people moving when they are 3 m from the right angle? (Round your answer to two decimal places.) 11) _____

A) 3.00 m/s B) 2.00 m/s C) 1.00 m/s D) 0.50 m/s

Solve the problem. Round your answer, if appropriate.

12) Water is discharged from a pipeline at a velocity v (in ft/sec) given by $v = 1054p^{(1/2)}$, where p is the pressure (in psi). If the water pressure is changing at a rate of 0.340 psi/sec, find the acceleration (dv/dt) of the water when $p = 57.0$ psi. 12) _____

A) 39.8 ft/sec^2 B) 1350 ft/sec^2 C) 23.7 ft/sec^2 D) 69.8 ft/sec^2

Solve the problem.

13) A container, in the shape of an inverted right circular cone, has a radius of 4 inches at the top and a height of 5 inches. At the instant when the water in the container is 2 inches deep, the surface level is falling at the rate of –1.3 in./s. Find the rate at which water is being drained. 13) _____

A) –10.45 $in.^3$/s B) –10.89 $in.^3$/s C) –14.64 $in.^3$/s D) –9.98 $in.^3$/s

Solve the problem. Round your answer, if appropriate.

14) One airplane is approaching an airport from the north at 150 km/hr. A second airplane approaches from the east at 297 km/hr. Find the rate at which the distance between the planes changes when the southbound plane is 27 km away from the airport and the westbound plane is 19 km from the airport. 14) _____

A) -588 km/hr B) -147 km/hr C) -441 km/hr D) -294 km/hr

15) A man 6 ft tall walks at a rate of 3 ft/sec away from a lamppost that is 13 ft high. At what rate is the length of his shadow changing when he is 50 ft away from the lamppost? (Do not round your answer) 15) _____

A) $\frac{9}{19}$ ft/sec B) $\frac{18}{7}$ ft/sec C) $\frac{18}{19}$ ft/sec D) 25 ft/sec

16) Electrical systems are governed by Ohm's law, which states that V = IR, where V = voltage, I = current, and R = resistance. If the current in an electrical system is decreasing at a rate of 4 A/s while the voltage remains constant at 14 V, at what rate is the resistance increasing (in Ω/sec) when the current is 44 A? (Do not round your answer.) 16) _____

A) $\frac{56}{11}$ Ω/sec B) $\frac{11}{14}$ Ω/sec C) $\frac{14}{11}$ Ω/sec D) $\frac{7}{242}$ Ω/sec

17) The volume of a rectangular box with a square base remains constant at 1000 cm^3 as the area of the base increases at a rate of 3 cm^2/sec. Find the rate at which the height of the box is decreasing when each side of the base is 11 cm long. (Do not round your answer.) 17) _____

A) $\frac{1000}{121}$ cm/sec B) $\frac{3}{121}$ cm/sec C) $\frac{3000}{1331}$ cm/sec D) $\frac{3000}{14641}$ cm/sec

18) The radius of a right circular cylinder is increasing at the rate of 4 in./sec, while the height is decreasing at the rate of 10 in./sec. At what rate is the volume of the cylinder changing when the radius is 19 in. and the height is 9 in.? 18) _____

A) -2926 $in.^3$/sec B) -2242π $in.^3$/sec C) 118 $in.^3$/sec D) -2926π $in.^3$/sec

Find the linearization L(x) of f(x) at x = a.

19) $f(x) = 3x^2 + 3x + 4$, a = 1 19) _____

A) L(x) = 3x + 7 B) L(x) = 3x + 1 C) L(x) = 9x + 7 D) L(x) = 9x + 1

20) $f(x) = \frac{1}{6x + 2}$, a = 0 20) _____

A) $L(x) = \frac{3}{2}x + \frac{1}{2}$ B) $L(x) = -\frac{3}{2}x + \frac{1}{4}$ C) $L(x) = \frac{3}{2}x + \frac{1}{4}$ D) $L(x) = -\frac{3}{2}x + \frac{1}{2}$

21) $f(x) = x + \frac{1}{x}$, a = 4 21) _____

A) $L(x) = \frac{17}{16}x + \frac{1}{2}$ B) $L(x) = \frac{15}{16}x + \frac{1}{2}$ C) $L(x) = \frac{15}{16}x + \frac{2}{5}$ D) $L(x) = \frac{17}{16}x + \frac{2}{5}$

22) f(x) = tan x, a = π 22) _____

A) L(x) = x – 3π B) L(x) = x – π C) L(x) = x + π D) L(x) = 3x – π

You want a linearization that will replace the function over an interval that includes the point x_0. To make your subsequent work as simple as possible, you want to center the linearization not at x_0 but at a nearby integer x = a at which the function and its derivative are easy to evaluate. What linearization do you use?

23) $f(x) = -3x^2 - 4x + 3$, $x_0 = 1.1$ 23) _____

A) -4 B) -6 - 10x C) 7 - 11x D) 6 - 10x

24) $f(x) = \sqrt{x}, x_0 = 9.1$ 24) _____

A) $\frac{3}{2} + \frac{1}{3}x$ B) $\frac{1}{2} + \frac{1}{2}x$ C) $\frac{3}{2} + \frac{1}{6}x$ D) $\frac{3}{2} - \frac{1}{6}x$

Use the linear approximation $(1 + x)^k \approx 1 + kx$, as specified.

25) Find an approximation for the function $f(x) = \frac{1}{\sqrt{2 + x}}$ for values of x near zero. 25) _____

A) $f(x) \approx \sqrt{2}\left(1 - \frac{x}{4}\right)$ B) $f(x) \approx 2 - \frac{x}{2}$

C) $f(x) \approx \frac{1}{\sqrt{2}}\left(1 - \frac{x}{4}\right)$ D) $f(x) \approx \frac{1}{\sqrt{2}}\left(1 - \frac{x}{2\sqrt{2}}\right)$

26) Find an approximation for the function $f(x) = (1 - x)^4$ for values of x near zero. 26) _____

A) $f(x) \approx 1 + 4x$ B) $f(x) \approx 4 + 4x$ C) $f(x) \approx 1 - 4x$ D) $f(x) \approx 1 + 5x$

27) Estimate $(1.0005)^{50}$. 27) _____

A) 1.005 B) 1.025 C) 1.05 D) 1.01

28) Estimate $\sqrt[3]{1.006}$. 28) _____

A) 1.02 B) 1.002 C) 1.003 D) 1.03

Find dy.

29) $y = 3x^2 + 6x + 8$ 29) _____

A) 6x dx B) 6x + 8 dx C) (6x + 6) dx D) 6x + 12 dx

30) $y = x\sqrt{9x - 6}$ 30) _____

A) $\frac{27x - 12}{\sqrt{9x - 6}}$ dx B) $\frac{27x - 12}{2\sqrt{9x - 6}}$ dx C) $\frac{27x + 12}{\sqrt{9x - 6}}$ dx D) $\frac{27x + 12}{2\sqrt{9x - 6}}$ dx

31) $6y^{1/2} - 3xy + x = 0$ 31) _____

A) $\left(\frac{3y - 1}{3y^{-1/2} + 3x}\right)$ dx B) $\left(\frac{3y - 1}{6y - 3x}\right)$ dx

C) $\left(\frac{3y - 1}{3y^{-1/2} - 3x}\right)$ dx D) $\left(\frac{-1}{3y^{-1/2} - 3x}\right)$ dx

32) $y = \cos(9\sqrt{x})$ 32) _____

A) $\left(\frac{-9\sqrt{x}\sin(9\sqrt{x})}{2}\right)$ dx B) $\left(\frac{-9\sin(9\sqrt{x})}{2\sqrt{x}}\right)$ dx

C) $\left(\frac{9\sqrt{x}\sin(9\sqrt{x})}{2}\right)$ dx D) $\left(\frac{9\sin(9\sqrt{x})}{2\sqrt{x}}\right)$ dx

The function f(x) changes value when x changes from x_0 to $x_0 + dx$. Find the approximation error $|\Delta f - df|$. Round your answer, if appropriate.

33) $f(x) = x^2$, $x_0 = 6$, $dx = 0.06$ 33) _____

A) 0.0036 B) 0.6636 C) 0.0018 D) 0.0072

34) $f(x) = x^3$, $x_0 = 4$, $dx = 0.05$ 34) _____

A) 0.0451875 B) 0.030125 C) 0.06025 D) 0.0150625

35) $f(x) = x - x^2$, $x_0 = 5$, $dx = 0.03$ 35) _____

A) 0.0549 B) 0.054 C) 0.108 D) 0.0009

Write a differential formula that estimates the given change in volume or surface area.

36) The change in the surface area $S = 4\pi r^2$ of a sphere when the radius changes from r_0 to $r_0 + dx$ 36) _____

A) $dS = 2\pi r_0\, dr$ B) $dS = 4\pi r_0^2\, dr$ C) $dS = 8\pi r_0\, dr$ D) $dS = 4\pi r_0\, dr$

Solve the problem.

37) A cube 4 inches on an edge is given a protective coating 0.3 inches thick. About how much coating should a production manager order for 700 cubes? 37) _____

A) About 3360 $in.^2$ B) About 13,440 $in.^3$

C) About 20,160 $in.^3$ D) About 10,080 $in.^2$

38) $A = \pi r^2$, where r is the radius, in centimeters. By approximately how much does the area of a circle decrease when the radius is decreased from 3.0 cm to 2.8 cm? (Use 3.14 for π.) 38) _____

A) 1.9 cm^2 B) 4.0 cm^2 C) 3.6 cm^2 D) 3.8 cm^2

39) The diameter of a tree was 11 in. During the following year, the circumference increased 2 in. About how much did the tree's diameter increase? (Leave your answer in terms of π.) 39) _____

A) $\frac{\pi}{2}$ in. B) $\frac{11}{\pi}$ in. C) $\frac{13}{\pi}$ in. D) $\frac{2}{\pi}$ in.

40) About how accurately must the interior diameter of a cylindrical storage tank that is 14 m high be measured in order to calculate the tank's volume within 4% of its true value? 40) _____

A) Within 4 meters B) Within 4%

C) Within 2 meters D) Within 2%

Use the differential to approximate the quantity to four decimal places.

41) $\sqrt{145}$ 41) _____

A) 12.0833 B) 12.0417 C) 13.0000 D) 11.9583

42) $e^{0.38}$ 42) _____

A) 1.4623 B) 1.3800 C) .6200 D) 0.6839

Find dy for the given values of x and Δx.

43) $y = x^3 + 2x$; $x = 2$, $\Delta x = 0.01$ 43) _____

A) 0.014 B) 0.14 C) 0.007 D) 0.07

44) $y = \frac{x^2}{\sqrt{x^2 + 21}}$; $x = 10$, $\Delta x = 0.1$ 44) _____

A) $\frac{146}{1331}$ B) $\frac{148}{1331}$ C) $\frac{144}{1331}$ D) $\frac{142}{1331}$

45) $y = 2x + 3$; $x = 18$, $\Delta x = 0.5$ 45) _____

A) 1 B) 0.5 C) 0.1 D) 5

Solve the problem.

46) A tumor is approximately spherical in shape. If the radius of the tumor changes from 8 mm to 10 mm, find the approximate change in volume. Round your answer to the nearest hundred. 46) _____

A) 2900 mm^3 B) 200 mm^3 C) 2100 mm^3 D) 1600 mm^3

Answer Key
Testname: CHAPTER 2 (PART III) RELATED RATES, DIFFERENTIALS, AND LINEAR APPROXIMATIONS

1) C
2) D
3) D
4) A
5) D
6) C
7) A
8) B
9) A
10) B
11) C
12) C
13) A
14) D
15) B
16) D
17) D
18) B
19) D
20) D
21) B
22) B
23) D
24) C
25) C
26) C
27) B
28) B
29) C
30) B
31) C
32) B
33) A
34) B
35) D
36) C
37) C
38) D
39) D
40) D
41) B
42) A
43) B
44) D
45) A
46) D